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Quantum Levitation of Graphene Sheet by Repulsive Casimir Forces

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We calculate the repulsive Casimir force between a single graphene sheet and a metamaterial slab whose magnetic permeability is expressed by using the Lorentz model, focusing on the possibility of realizing quantum levitation. It is shown that the graphene sheet can be levitated by the repulsive Casimir force because of the very small density of the graphene sheet. The levitation height depends on the magnetic properties of the metamaterial slab, and this height reaches the micrometer range when metamaterials with large permeabilities are used. The possibility of observing quantum levitation in vacuum is also considered.

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I. INTRODUCTION

Quantum levitation by Casimir forces [1–3] in atmosphere has attracted our attention for the last several years because this phenomenon allows us to explore new possibilities involving superlubricity and the "antistiction" effect [4, 5]. However, these possibilities have not been observed yet. The aim of this study is to show that a combination of a graphene sheet and a magnetodielectic metamaterial [6] is suitable for the verification of quantum levitation.

The Casimir force between metallic bodies has already been measured with high precision [7, 8], and this force is always attractive irrespective of the separation between the metallic bodies. For the possibility of the repulsive Casimir force, a no-go theorem is known. Kenneth and Klich proved that the Casimir interaction between two (non-magnetic) dielectric bodies related by reflection is always attractive, independently of the exact form of the bodies or their dielectric properties [9, 10]. This result implies that it is difficult to generate a repulsive Casimir force. One of the methods to realize the repulsive Casimir force is to use of metamaterials.

Recent theoretical studies on repulsive Casimir forces acting on metamaterial slabs suggest that the magnitude of repulsive Casimir forces generated using existing metamaterials is weak compared with that of attractive Casimir forces between metallic plates measured previously [11–13]. This suggests that a very light material is necessary to observe the quantum levitation using current technology.

The magnitude of the Casimir force between dielectric bodies depends on the shape of the bodies. The configuration usually used in experiments is a combination of a sphere and a plate because the gap between them can be easily controlled. Although the Casimir force per unit area between parallel plates is much stronger than that between a sphere and a plate with the same separation, we encounter many difficulties in the measurement of the Casimir force between parallel plates [14]. The difficulties arise primarily from two technical problems. First, it is difficult to precisely position two large rigid plates in parallel. Second, the roughness of the surface generates fluctuations in the Casimir force.

A graphene sheet is very light and flat. It is also strong but flexible [15]. These properties can be used to overcome the abovementioned technical problems. However, a possible disadvantage is that the graphene sheet is very thin. The Casimir force between macroscopic plates with finite thickness decreases with the thickness. If we assume that this behavior is valid for atomic scales, the Casimir force acting on the graphene sheet is negligibly small. However, this assumption is not true. Bordag *et al.* calculated the Casimir force between a graphene sheet and gold and found that it is approximately 45% of the Casimir force between ideal metals at 1 μm separation [16]. We calculate the Casimir force between the graphene sheet and a metamaterial slab on the basis of the formula developed by Bordag *et al.* and determine the position at which the graphene sheet is stably levitated from the metamaterials.

II. METHOD OF CALCULATION

Before we derive the Casimir force between an infinite graphene sheet and an infinite slab made of metamaterials, it would be useful to discuss the Casimir force between two perfectly conductive plates. The Casimir force originates from the fluctuations of an electromagnetic field. Thus, the optical properties of the materials determine the magnitude of the Casimir force. Lights cannot penetrate perfectly conductive plates. Hence, the allowed frequencies depend on the separation between the plates; the zero-point energy changes depending on the separation

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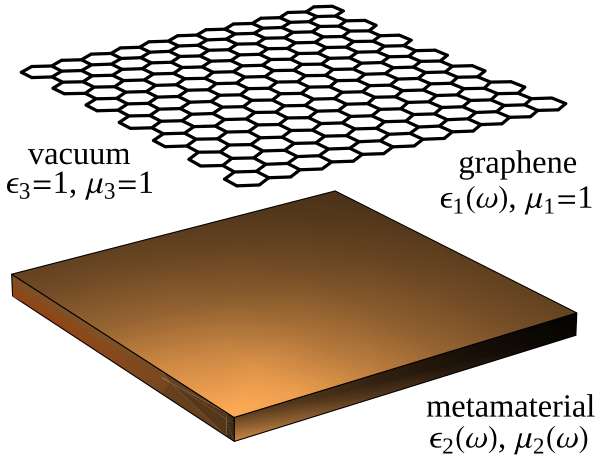

FIG. 1: Configuration of graphene sheet and metamaterial slab. The graphene sheet is laid in parallel to a metamaterial slab with a separation a.

between the perfectly conductive plates. H. G. Casimir showed the force between the perfectly conductive plates is attractive, and its magnitude per unit area is given by

$$F_C(a) = -\frac{\pi^2 \hbar c}{240 a^4}, \tag{1}$$

where a is the separation between the plates [1]. The force is evaluated as $-1.3/a^4$ mPa (a in micrometers). Accordingly, the Casimir force per 1 m^2 between perfectly conductive plates with $a = 1$ μm corresponds to the gravitational attraction between 0.13 g of mass and the earth.

Now, we consider the Casimir force between an infinite graphene sheet and an infinite slab made of metamaterials. The configurations of the sheet and the slab are shown in Fig. 1.

According to usage, the graphene and the metamaterial slab are labeled as 1 and 2, respectively, and the vacuum space between them is labeled as 3. Let the reflection coefficients of the material labeled as i in the TM and TE modes be $r^{(i)}_{\mathrm{TM}}(\omega)$ and $r^{(i)}_{\mathrm{TE}}(\omega)$, respectively. Then, the Casimir force per unit area, which is derived by Lifshitz [17], is given by

$$F(a) = -\frac{\hbar}{2\pi^2}\int_0^\infty K_3 k_\perp dk_\perp \int_0^\infty d\xi \left(\frac{r^{(1)}_{\mathrm{TM}} r^{(2)}_{\mathrm{TM}}}{e^{2aK_3} - r^{(1)}_{\mathrm{TM}} r^{(2)}_{\mathrm{TM}}} + \frac{r^{(1)}_{\mathrm{TE}} r^{(2)}_{\mathrm{TE}}}{e^{2aK_3} - r^{(1)}_{\mathrm{TE}} r^{(2)}_{\mathrm{TE}}} \right), \tag{2}$$

where $k_\perp$ is the magnitude of the wave vector parallel to the surface, ξ is the frequency variable along the imaginary axis ($\omega = i\xi$), and K_3 is defined as

$$K_3 = \sqrt{k_\perp^2 + \frac{\xi^2}{c^2}}. \tag{3}$$

If the thickness of the metamaterial slab is infinite, its reflection coefficients are given by

$$r^{(2)}_{\mathrm{TM}} = \frac{\epsilon_2(\omega) K_3 - K_2}{\epsilon_2(\omega) K_3 + K_2}, \qquad r^{(2)}_{\mathrm{TE}} = \frac{K_2 - \mu_2(\omega) K_3}{K_2 + \mu_2(\omega) K_3}, \tag{4}$$

where $\epsilon_i(\omega)$ and $\mu_i(\omega)$ are the dielectric function and the magnetic permeability of the metamaterial slab. Here, the function K_i for $i = 1, 2$ is defined as

$$K_i = \sqrt{k_\perp^2 + \epsilon_i(\omega)\mu_i(\omega)\frac{\xi^2}{c^2}}. \tag{5}$$

Rosa *et al.* generalized the Lifshitz theory and calculated the Casimir force between dielectric bodies and metamaterials. Because typical metamaterials have resonant electromagnetic response at certain frequencies, they employed a Drude-Lorentz model to express the dielectric function and the magnetic permeability of a metamaterial slab as a function of the wave frequency [18]. We also use the same functions in this study, these functions are defined as

$$\epsilon_2(\omega) = 1 - \frac{\Omega_e^2}{\omega^2 - \omega_e^2 + i\gamma_e\omega}, \qquad \mu_2(\omega) = 1 - \frac{\Omega_m^2}{\omega^2 - \omega_m^2 + i\gamma_m\omega}. \tag{6}$$

The parameters are given by $\Omega_e = 2.71\omega_g$, $\Omega_m = 6.77\omega_g$, $\omega_e = \omega_m = 6.77\omega_g$, $\gamma_e = \gamma_m = 0.34\omega_g$ [11]. Here the wave frequency $\omega_g \equiv 2.02 \times 10^{14}$ is a very important value in this study, and we provide details with regard to this frequency in subsequent paragraph.

We turn now to consideration of the reflection coefficients of a graphene sheet. In the reference [16], the reflection coefficients of a graphite plate of thickness d is given by

$$r_{\mathrm{TM}}^{\mathrm{graphite}} = \frac{\epsilon_x(i\xi)\epsilon_z(i\xi)K_3^2 - k_z^2}{\epsilon_x(i\xi)\epsilon_z(i\xi)K_3^2 + k_z^2 + 2K_3k_z\sqrt{\epsilon_x(i\xi)\epsilon_z(i\xi)}\coth(k_zd)},$$
$$r_{\mathrm{TE}}^{\mathrm{graphite}} = \frac{k_x^2 - K_3^2}{k_x^2 + K_3^2 + 2K_3k_x\coth(k_xd)}, \quad (7)$$

where $\epsilon_x(\omega) = \epsilon_y(\omega)$ and $\epsilon_z(\omega)$ are the graphite dielectric permittivity in the directions x, y, and z, respectively, and $k_q = \sqrt{k_\perp^2 + \epsilon_q(i\xi)\frac{\xi^2}{c^2}}$ for $q = x, y$. It should be noted that the reflection coefficients of a graphene sheet are not obtained from eq. (7) in the limit $d \to 0$.

Bordag *et al.* regarded a graphene sheet as a plasma sheet and derived the reflection coefficients from marching conditions on the tangential and normal component of the electromagnetic field. The obtained reflection coefficients for the graphene sheet are

$$r_{\mathrm{TM}}^{(1)} = \frac{c^2\kappa_gK_3}{c^2\kappa_gK_3 + \xi^2},$$
$$r_{\mathrm{TE}}^{(1)} = \frac{\kappa_g}{\kappa_g + K_3}, \quad (8)$$

where $\kappa_g = \omega_g^{-1}$. The quantity κ_g, which characterizes the wave length for the electromagnetic field, is given by

$$\kappa_g = 2\pi\frac{ne^2}{mc^2}, \quad (9)$$

where $n = 3.81\times10^{19}/\mathrm{m}^2$ denotes the density of electrons. By submitting the reflection coefficients given by eqs. (4) and (8) in eq. (2), we can obtain the Casimir force between a graphene sheet and a metamaterial slab.

III. RESULTS AND DISCUSSION

The levitation of a graphene sheet can be realized if the Casimir force between a graphene sheet and a metamaterial slab balances the force of gravity. We calculate the gravitational attraction acting on a graphene sheet per unit area. The minimum distance between carbon atoms in a six-membered ring is 1.412 Å. Thus, the area occupied by the six-membered ring of carbon atoms is 5.180 Å^2. Because two atoms are distributed to each area occupied by a six-membered ring in a graphene sheet, the mass of a graphene sheet per unit area is estimated as $\rho = 2m_c/(5.180\ \text{Å}^2) = 0.769\ \mathrm{mg/m^2}$, where $m_c = 1.993 \times 10^{-26}$ kg denotes the mass of a carbon atom. Accordingly, the magnitude of the gravitational force acting on a graphene sheet per unit area is $F_G = 9.80\rho = 7.54\ \mu\mathrm{N/m^2}$.

Figure 2 shows the Casimir force between the graphene sheet and the metamaterial slab per unit area for different magnetic resonance frequencies in combination with a plot of the gravitational force. Both forces are normalized by the Casimir force between perfectly conductive plates given by eq. (1). We append the sign to this ratio of the Casimir forces so as to take negative value for the repulsive force. As can be observed from Fig. 2, the attractive Casimir force changes to a repulsive one as the separation increases.

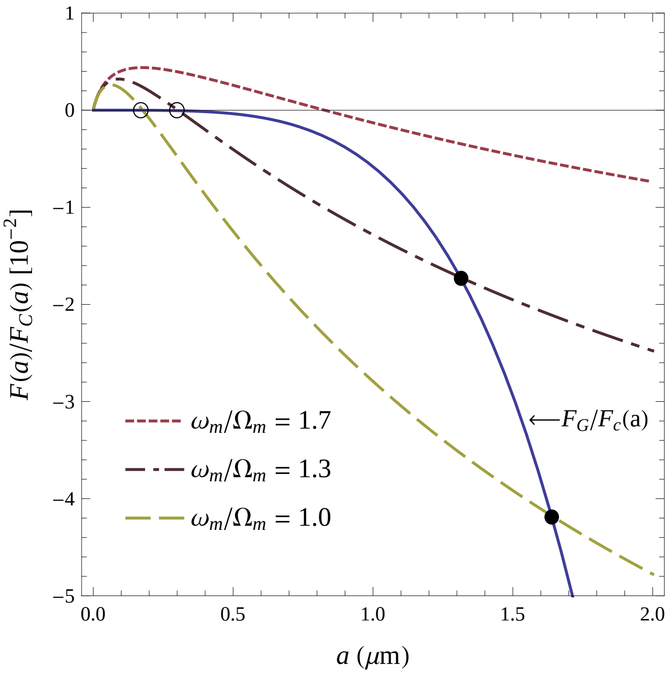

FIG. 2: Casimir force between graphene sheet and metamaterial slab for different magnetic resonance frequencies ω_m/Ω_m=1.0, 1.3 and 1.7. The gravitational force acting on the graphene sheet scaled the Casimir force between perfectly conductive plates is represented by a solid line. The intersections of the dashed lines and the solid line indicate the separation at equilibrium states. The solid (open) circles denote the stable(unstable) equilibrium positions and the Casimir forces at those positions.

The gravitational force is indicated by a solid line and its sign is set to be negative so as to allow its comparison with the repulsive Casimir force easily. When $\omega_m/\Omega_m = 1.7$, the solid line does not intersect the shortdashed line. This implies that the repulsive Casimir force is insufficient to levitate the graphene sheet. As the value of ω_m is decreased, the solid line intersects the dashed line at two points, which indicate the equilibrium positions. The graphene sheet can levitate only at larger separation values because the equilibrium point at shorter separation values is unstable.

Figure 3(a) shows the height to which the graphene sheet is levitated from the surface of the metamaterial slab as a function of ω_m. The height of the stable equilibrium position increases as the value of ω_m decreases. The levitation can be observed only below a critical value of $\omega_m/\Omega_m = 1.57$. The static magnetic permeability modeled by using the Drude-Lorentz model $\mu(0)$ is given by $1+\Omega_m^2/\omega_m^2$. Hence the decrease in the frequency ω_m leads to the increase in the static magnetic permeability. The static magnetic permeability at the critical value is 1.41. Although the magnitude of the repulsive Casimir force is not determined solely by the static magnetic permeability, it offers us important information. A simple method to increase the static magnetic permeability is to increase Ω_m. Figure 3(b) shows the levitation height as a function of Ω_m. As expected, the levitation height increases with Ω_m.

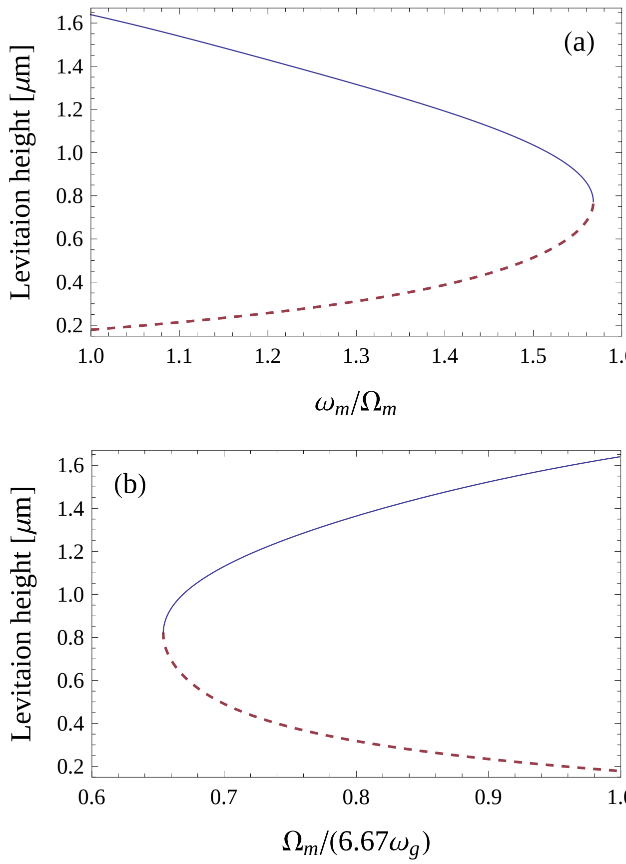

FIG. 3: Plots of separations between levitated graphene sheet and surface of metamaterial slab as functions of (a) magnetic resonance frequency of metamaterial slab and (b) Drude-Lorentz parameter Ω_m. The solid and dashed lines indicate the separation length at stable and unstable levitations, respectively.

IV. CONCLUSION

In this study, we analyzed the levitation of a graphene sheet by repulsive Casimir forces. Although more extensive studies are required to determine the repulsive Casimir force preciously, our results showed that the graphene sheet is a good candidate for the demonstration of quantum levitation. Graphene sheets are probably the flattest [19] and lightest materials available today and hence are considered to be most suitable for the demonstration of quantum levitation. Therefore, the question to be addressed is whether the magnitude of the repulsive Casimir force acting on the graphene sheet is sufficiently large to levitate the graphene sheet. By combining the results of the studies conducted by Rosa *et al.* and Bordag *et al.*, we showed that the repulsive Casimir force can levitate the graphene sheet for possible metamaterial slab. Although we could not propose the use of a specific material, the parameters used in our calculation are not unphysical values. Thus it would be beneficial to develop metamaterial slabs having a dielectric function and magnetic permeability same as those obtained in this study.

We have to seek for mainly magnetic material in order to realize the quantum levitation. Pirozhenko and Lambrecht have suggested that array of dielectric spheres in a dielectric matrix with the magnetic response arising from poalritonic resonances is a prospective metamaterial [12]. Schller *et al.* have demonstrated that dielectric metamaterial made of silicon carbide particle exhibits both electric and magnetic optical resonances [6, 18]. In addition, since the repulsive Casimir force decreases rapidly as the separation increases, it is desirable that metamaterials inherit magnetic response in the visible range of the electromagnetic spectrum. Photonic metamaterials with negative-index at 780 nm wavelength are already demonstrated [20]. In the near future, we have to design optimal metamaterials that can produce strong repulsive Casimir force by tuning these magnetic response [21, 22].

We intend to improve our theoretical analysis in further studies. As has been pointed out by Rosa *et al.*, the anisotropy of the metamaterial slab must be considered. Furthermore, the analysis of the graphene sheet must be extended beyond plasma sheet model. Recently, Fialkosy studied the Casimir interaction between suspended graphene films and an ideal conductor using the Dirac model [23].

Developing a method that can be used to observe the levitation of graphene sheets is a future challenging. A very simple method to confirm levitation is to observe the movement of the graphene sheet while tilting the metamaterials, if the superlubricity of the graphene sheet can be achieved.

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